

## Sheep and Ducks



The farmer raises sheep and ducks.

1. How many legs on one duck? \_\_\_\_\_
2. How many legs on 4 ducks? \_\_\_\_\_
3. How many legs on 5 sheep? \_\_\_\_\_

4. Next to the barn is a pen with 2 sheep and 3 ducks. How many legs altogether? \_\_\_\_\_

Show how you know your answer is correct.

5. One of the farmer's pens has a high fence around it. He can see 32 legs under the fence. How many sheep and ducks are in this pen?

Show one way to have sheep and ducks with 32 legs in all.

Show a different number of sheep and ducks with 32 legs in all.

# Sheep and Ducks

## Mathematics Assessment Collaborative

### Performance Assessment Rubric Grade 2

	<b>Task 1: Sheep and Ducks: Grade 2:</b>	Points	Section Points
	The core elements of the performance required by this task are: <ul style="list-style-type: none"> <li>• Demonstrate fluency in adding and subtracting whole numbers</li> <li>• Understand situations that entail multiplication such as equal groupings of objects</li> <li>• Describe and extend patterns of number and translate from one representation to another</li> <li>• Communicate reasoning using words, numbers or pictures</li> </ul>		
1	Gives correct answer as: <b>2</b>	<b>1</b>	<b>1</b>
2	Gives correct answers as: <b>8</b>	<b>1</b>	<b>1</b>
3	Gives correct answer as: <b>20</b>	<b>1</b>	<b>1</b>
4	Gives a correct answer as: <b>14</b>  Shows work such as: $4 + 4 + 2 + 2 + 2$	<b>1</b>  <b>1</b>	<b>2</b>
5	Gives a correct answer from the following:  16 ducks 14 ducks, 1 sheep 12 ducks, 2 sheep 10 ducks, 3 sheep 8 ducks, 4 sheep 6 ducks, 5 sheep 4 ducks, 6 sheep 2 ducks, 7 sheep 8 sheep  Shows a different correct answer from those listed above	<b>1</b>  <b>2</b>	<b>3</b>
	<b>Total Points</b>		<b>8</b>

## 2<sup>nd</sup> grade – Task 1: Sheep and Ducks

Work the task and examine the rubric.

What do you think are the key mathematics the task is trying to assess?

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Look at the student work in parts 1, 2 and 3:

What successful strategies did students use?

Just seemed to know	Drew pictures and counted	Used repeated addition	Used multiplication facts	Other

How might these successful strategies be shared to increase success throughout the class?

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Look at the student work for part 4:

What strategies did students use to solve:

Drew pictures and counted	$4 + 4 + 2 + 2 + 2$	$8 + 6$	Other	Errors

When you look at the papers with errors in part 4 – what similar ones were being made?  
What are the implications for instruction?

Look at the student work for part 5:

What successful strategies did students use to find 32 legs in all?

Drew pictures	Used tallies or circles	Used addition or subtraction	Skip counted	Used a known fact	Other

How might these strategies be shared to focus all on success?

Look at the errors in part 5:

What types of errors are being made?

Can you sort the errors into different categories?

What are the implications for future instruction?

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## Looking at Student Work on Sheep and Ducks

Most children pass through three strategy levels in acquiring problem solving skills. These first three papers are examples of each of these phases. Students A – C all have tools available for their use in solving problems. Student A uses direct modeling of the actions in the task through the use of pictures of the animals and representations of their legs.



### Student A




## Ovejas y Patos



El agricultor cría ovejas y patos.

- ¿Cuántas piernas hay en un pato? dos ✓
- ¿Cuántas piernas en 4 patos? ocho ✓ 
- ¿Cuántas piernas en 5 ovejas? veinte ✓ 


4. Al lado del establo hay un redil con 2 ovejas y 3 patos.  
¿Cuántas piernas hay en total? catorce ✓ 

Muestra cómo sabes que tu respuesta es correcta. ✓

5. Uno de los rediles tiene una cerca alta alrededor. Él puede ver 32 piernas por debajo de la cerca. ¿Cuántas ovejas y patos hay en el redil?  
Muestra una manera de tener ovejas y patos con 32 piernas en total.

8 patos  
4 ovejas ✓

Muestra un número diferente de ovejas y patos con 32 piernas en total.

 ✓


6 ovejas    4 patos

⑤


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Student B uses an additive strategy to successfully solve each part of the task. Look closely at the work in the 2<sup>nd</sup> part of 5. Notice how the student changes the adding process. What evidence do we have of understanding?

**Student B**



### Sheep and Ducks



The farmer raises sheep and ducks.

1. How many legs on one duck? 2 ✓  $1+1=2$
2. How many legs on 4 ducks? 8 ✓  $4 \times 2 = 8$
3. How many legs on 5 sheep? 20 ✓  $5 \times 4 = 20$

4. Next to the barn is a pen with 2 sheep and 3 ducks. How many legs altogether? 14 ✓  $4+4+2+2+2=14$

Show how you know your answer is correct.

5. One of the farmer's pens has a high fence around it. He can see 32 legs under the fence. How many sheep and ducks are in this pen?  $4+2+2+4+4+2+2+4+4+2$

Show one way to have sheep and ducks with 32 legs in all.  $=32$

Sheep	<u>5</u>	$4+4+4+4+4=20$ ✓
Duck	<u>6</u>	$2+2+2+2+2+2=12$
		$32$

Show a different number of sheep and ducks with 32 legs in all.



Sheep	<u>3</u>	$2+2+2+2+2+2=12$ ✓
Duck	<u>10</u>	$4+4+4+4+4=20$ ✓
		$32$

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Student C has access through number facts that are easily recalled.

Student C

 **Sheep and Ducks** 

The farmer raises sheep and ducks.

1. How many legs on one duck? 2 ✓

2. How many legs on 4 ducks? 8 ✓

3. How many legs on 5 sheep? 20 ✓

4. Next to the barn is a pen with 2 sheep and 3 ducks. How many legs altogether? 14 ✓

Show how you know your answer is correct.

$4 + 4 = 8$   $2 + 2 = 4$   
 $8 + 2 = 10$   $10 + 4 = 14$

5. One of the farmer's pens has a high fence around it. He can see 32 legs under the fence. How many sheep and ducks are in this pen?

Show one way to have sheep and ducks with 32 legs in all.

$14$  ducks +  $18$  sheep =  $32$

Show a different number of sheep and ducks with 32 legs in all.

$12$  ducks +  $20$  sheep =  $32$

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Students D – H are in different stages of direct modeling. Student D, below, still needs to draw replicas of the animals but is moving toward representations of the ducks toward the end of the task. *What is the evidence that Student D is still counting by ones?*

**Student D**



**Sheep and Ducks**



The farmer raises sheep and ducks.

1. How many legs on one duck? 2 legs
2. How many legs on 4 ducks? 8 legs
3. How many legs on 5 sheep? 20 legs

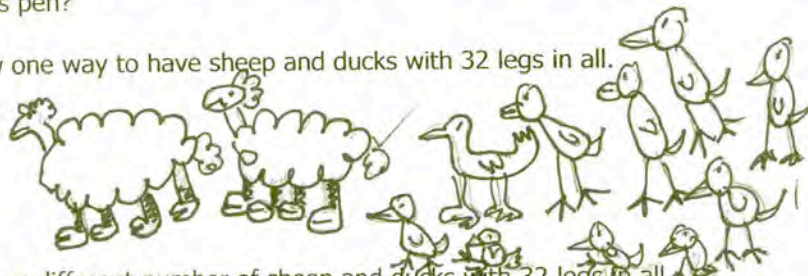
4. Next to the barn is a pen with 2 sheep and 3 ducks. How many legs altogether? 14 legs

Show how you know your answer is correct.



5. One of the farmer's pens has a high fence around it. He can see 32 legs under the fence. How many sheep and ducks are in this pen?

Show one way to have sheep and ducks with 32 legs in all.



Show a different number of sheep and ducks with 32 legs in all.





Student E has moved to a slightly abstracted form of the animals but we can see no evidence of more than the counting of each leg to find a grand total.

**Student E**




**Sheep and Ducks**

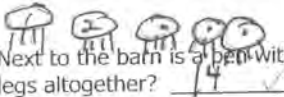


The farmer raises sheep and ducks.

1. How many legs on one duck? 2 ✓

2. How many legs on 4 ducks? 8 ✓ 

3. How many legs on 5 sheep? 20 ✓

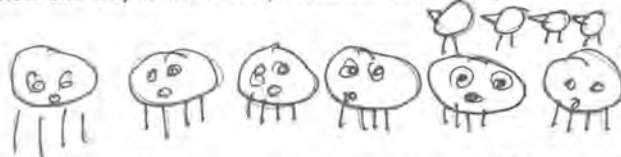
4. Next to the barn is a pen with 2 sheep and 3 ducks. How many legs altogether? 14 ✓ 

Show how you know your answer is correct.

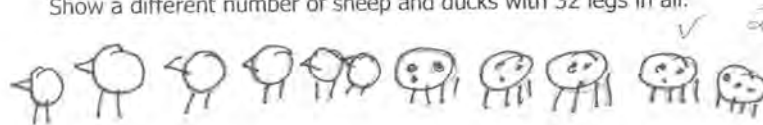


5. One of the farmer's pens has a high fence around it. He can see 32 legs under the fence. How many sheep and ducks are in this pen?

Show one way to have sheep and ducks with 32 legs in all.



Show a different number of sheep and ducks with 32 legs in all.



Look at the organizational system used by Student F. What about this system might help other student in your class? What makes this system efficient? What questions might you ask to re-engage this student in the error in the 2<sup>nd</sup> part of 5?

**Student F**



**Sheep and Ducks**



The farmer raises sheep and ducks.

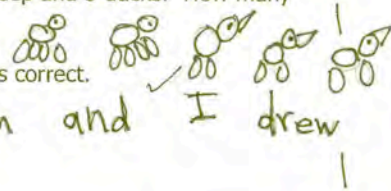
1. How many legs on one duck? 2 ✓
2. How many legs on 4 ducks? 8 ✓
3. How many legs on 5 sheep? 20 ✓



4. Next to the barn is a pen with 2 sheep and 3 ducks. How many legs altogether? 14 ✓

Show how you know your answer is correct.

I count them and I drew them.



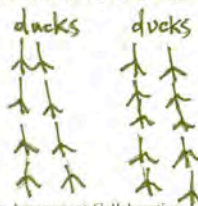
5. One of the farmer's pens has a high fence around it. He can see 32 legs under the fence. How many sheep and ducks are in this pen?

Show one way to have sheep and ducks with 32 legs in all.



I add 6 sheep and add 4 ducks.

Show a different number of sheep and ducks with 32 legs in all.



I add 9 of sheep and add 3 of the duck..

0

6

Student G has moved to a completely representational modeling of the situation in this task. *What part of this task would you like to know more about? Why? What questions might prompt the student to refocus on this error?*

**Student G**



**Sheep and Ducks**



The farmer raises sheep and ducks.

1. How many legs on one duck?  $\frac{2}{8}$  ✓
2. How many legs on 4 ducks?  $\frac{8}{20}$  ✓
3. How many legs on 5 sheep?  $\frac{20}{20}$  ✓

4. Next to the barn is a pen with 2 sheep and 3 ducks. How many legs altogether?  $\frac{14}{14}$  ✓

Show how you know your answer is correct.



5. One of the farmer's pens has a high fence around it. He can see 32 legs under the fence. How many sheep and ducks are in this pen?

Show one way to have sheep and ducks with 32 legs in all.




Show a different number of sheep and ducks with 32 legs in all.




Student H directly models using shapes similar to sheep and duck footprints. There is evidence of the ability to show the same answer in different representations. The error in part 4 was a common misunderstanding. *Given the rest of the evidence of understanding, what would you suggest to reassess on this question?*

**Student H**



### Sheep and Ducks



The farmer raises sheep and ducks.

- How many legs on one duck?  $\underline{2}$  ✓
- How many legs on 4 ducks?  $\underline{8}$  ✓
- How many legs on 5 sheep?  $\underline{20}$  ✓

4. Next to the barn is a pen with 2 sheep and 3 ducks. How many legs altogether?  $\underline{5}$  X

$2+3=5$   
~~X~~

Show how you know your answer is correct.

5. One of the farmer's pens has a high fence around it. He can see 32 legs under the fence. How many sheep and ducks are in this pen?

$\circ\circ\circ\circ$   $\circ\circ\circ\circ$   $\circ\circ\circ\circ$   $\circ\circ\circ\circ$   $\circ\circ\circ\circ$   
 $\Delta\triangledown$   $\Delta\triangledown$   $\Delta\triangledown$   $\Delta\triangledown$   $\Delta\triangledown$   $\Delta\triangledown$

Show one way to have sheep and ducks with 32 legs in all.

5 sheep  
 6 ducks

Show a different number of sheep and ducks with 32 legs in all.

$\frac{4}{2}$   $\frac{4}{2}$   $\frac{4}{2}$   $\frac{4}{2}$   $\frac{4}{2} \times 2$

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Gradually, students replace direct models with more efficient counting strategies. Students I-M are using counting and adding strategies to solve this task.

Look at the work of Student I. *What addition strategies are flexibly integrated in this child's "toolbox"?*

**Student I**



**Sheep and Ducks**



The farmer raises sheep and ducks.

1. How many legs on one duck?  $\underline{2}$  ✓✓
2. How many legs on 4 ducks?  $\underline{8}$  ✓✓
3. How many legs on 5 sheep?  $\underline{20}$  ✓✓

11

4. Next to the barn is a pen with 2 sheep and 3 ducks. How many legs altogether?  $\underline{14}$  ✓✓

11

Show how you know your answer is correct.

$4+4=8$   
 $2+2+2=6$   
 $8+6=14$

Sheeps      Ducks

5. One of the farmer's pens has a high fence around it. He can see 32 legs under the fence. How many sheep and ducks are in this pen?

Show one way to have sheep and ducks with 32 legs in all.

$6+6=12$  Ducks  
 $12+6=18$   
 $12+4=16$

$6+6=12$  Sheep  
 $8+8=16$

Show a different number of sheep and ducks with 32 legs in all.

Duck legs = 8  
 $4+4=8$

$24 =$  Sheep legs  
 $8+8+8=24$



Student J understands what it can mean to show how an answer is correct. Using a subtraction process for finding the number and kind of animals with 32 legs in all, he moves fluently between subtracting one animal at a time to sometimes, when more efficient, subtracting one of each type of animal together (- 6 legs).

**Student J**



**Sheep and Ducks**



The farmer raises sheep and ducks.

1. How many legs on one duck? 2 ✓
2. How many legs on 4 ducks? 8 ✓
3. How many legs on 5 sheep? 24 ✗

4. Next to the barn is a pen with 2 sheep and 3 ducks. How many legs altogether? 14 ✓

Show how you know your answer is correct.

$$\begin{array}{l} \text{||||} \\ \text{||||} = 8 + 6 = \text{||||} \quad \checkmark \\ \text{Sheep legs} \quad \text{duck legs} \\ \quad \quad \quad \downarrow \quad \downarrow \\ \quad \quad \quad 14 \quad 12 \end{array}$$

5. One of the farmer's pens has a high fence around it. He can see 32 legs under the fence. How many sheep and ducks are in this pen?

Show one way to have sheep and ducks with 32 legs in all.

$$\begin{array}{l} \text{4 ducks} \\ \text{8 legs} \\ \text{6 sheeps} \\ \text{24 legs} \\ \text{10 animals} \end{array}$$

$$\begin{array}{l} \text{ducks} \text{ ||||| } 32 - 2 = 30 - 4 = 26 - 4 = 22 - 2 = 20 \\ \text{sheeps} \text{ |||| } 20 - 4 = 16 - 2 = 14 - 4 = 10 - 2 = 8 \\ \quad \quad \quad 8 - 4 = 4 - 4 = 0 \end{array}$$

4 ducks = 8 legs sheeps = 24

Show a different number of sheep and ducks with 32 legs in all.

$$\begin{array}{l} \text{ducks} \text{ ||||| } 32 - 4 = 28 - 6 = 22 - 2 = 20 \\ \text{sheps} \text{ |||| } 20 - 4 = 16 - 6 = 10 - 2 = 8 - 8 = 0 \\ \quad \quad \quad \downarrow \quad \downarrow \\ \quad \quad \quad 8 \text{ ducks} = 16 \text{ legs} \\ \quad \quad \quad 4 \text{ sheps} = 16 \text{ legs} \end{array}$$

12 animals

⑦

Student K uses skip counting and addition to show proof and justify her answer.

Student K



### Sheep and Ducks



The farmer raises sheep and ducks.

- How many legs on one duck?  $\underline{2}$
- How many legs on 4 ducks?  $\underline{8}$
- How many legs on 5 sheep?  $\underline{20}$

4. Next to the barn is a pen with 2 sheep and 3 ducks. How many legs altogether?  $\underline{14}$

Show how you know your answer is correct.

<p>sheep</p> $\begin{array}{c} 4 \\ 8 \\ \hline 8 \end{array}$	<p>ducks</p> $\begin{array}{c} 2 \\ 4 \\ 6 \\ \hline 6 \end{array}$	$\begin{array}{c} 11 \\ 11 \\ \hline 22 \\ 14 \end{array}$
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5. One of the farmer's pens has a high fence around it. He can see 32 legs under the fence. How many sheep and ducks are in this pen?

Show one way to have sheep and ducks with 32 legs in all.

<p>5 sheep</p> $\begin{array}{c} 4 \\ 8 \\ 12 \\ 16 \\ 20 \\ \hline 20 \end{array}$	<p>6 ducks</p> $\begin{array}{c} 2 \\ 4 \\ 6 \\ 8 \\ 10 \\ 12 \\ \hline 12 \end{array}$	$\begin{array}{r} 12 \\ + 20 \\ \hline 32 \end{array}$
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Show a different number of sheep and ducks with 32 legs in all.


<p>7 sheep</p> $\begin{array}{c} 4 \\ 8 \\ 12 \\ 16 \\ 20 \\ 24 \\ 28 \\ \hline 28 \end{array}$	<p>ducks</p> $\begin{array}{c} 2 \\ 4 \\ \hline 4 \end{array}$	$\begin{array}{r} 28 \\ + 4 \\ \hline 32 \end{array}$
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Student L uses skip counting in a flexible manner. He is able to count by 4s and add on from a subtotal of 28 by 2s.

**Student L**



## Sheep and Ducks



The farmer raises sheep and ducks.


- How many legs on one duck? 2 ✓
- How many legs on 4 ducks? 8 ✓
- How many legs on 5 sheep? 20 ✓  $4+4+4+4+4=20$

4. Next to the barn is a pen with 2 sheep and 3 ducks. How many legs altogether? 14 ✓

Show how you know your answer is correct.

Ducks  $||||$  ✓

Sheep  $|||||$



5. One of the farmer's pens has a high fence around it. He can see 32 legs under the fence. How many sheep and ducks are in this pen?

Sheep 4 8 12 16 20 24 28 32 =

Show one way to have sheep and ducks with 32 legs in all. ✓

0 Ducks 32

32 Sheep

legs |

Show a different number of sheep and ducks with 32 legs in all.

4 8 12 16 20 24 28 30 32

Sheep                  ✓ Ducks

2

(7, 8)



What strategies are evidenced in the work of Student M? What error do you find in the 2<sup>nd</sup> part of 5? How might you handle a re-engagement with this student?

Student M



### Sheep and Ducks



The farmer raises sheep and ducks.

1. How many legs on one duck? 2 ✓
2. How many legs on 4 ducks? 8 ✓
3. How many legs on 5 sheep? 20 ✓

4. Next to the barn is a pen with 2 sheep and 3 ducks. How many legs altogether? 14 ✓

Show how you know your answer is correct.

$$4 + 4 + 2 + 2 + 2 = 14$$

5. One of the farmer's pens has a high fence around it. He can see 32 legs under the fence. How many sheep and ducks are in this pen?

Show one way to have sheep and ducks with 32 legs in all.

~~44444444~~ ✓

Show a different number of sheep and ducks with 32 legs in all.

~~444444~~ 2222 ✓

6

Students N and O are using number facts to solve problems but with different degrees of success. What understandings do both students show? Why might the scorer have marked Student O's explanation in part 4 incorrect? So you agree? Why or why not?

**Student N**



**Sheep and Ducks**



The farmer raises sheep and ducks.

1. How many legs on one duck? 2 ✓
2. How many legs on 4 ducks? 8 ✓
3. How many legs on 5 sheep? 20 ✓

4. Next to the barn is a pen with 2 sheep and 3 ducks. How many legs altogether? 14 ✓

Show how you know your answer is correct.

1 sheep = 4 legs     $2 \times 4 = 8$     1 duck = 2 legs

$2 \times 3 = 6$      $8 + 6 = 14$

5. One of the farmer's pens has a high fence around it. He can see 32 legs under the fence. How many sheep and ducks are in this pen? 2 ducks 7 sheep ✓

Show one way to have sheep and ducks with 32 legs in all.

$2 \text{ ducks} = 4 \text{ legs}$  /  $7 \text{ sheep} = 28 \text{ legs}$   
 $28 + 4 = 32$

Show a different number of sheep and ducks with 32 legs in all.

$4 \text{ ducks} = 8 \text{ legs}$  /  $6 \text{ sheep} = 24 \text{ legs}$  ✓  
 $24 + 8 = 32$

(8)

Student O



Sheep and Ducks



The farmer raises sheep and ducks.

1. How many legs on one duck? 2 ✓
2. How many legs on 4 ducks? 8 ✓
3. How many legs on 5 sheep? 20 ✓

$$2 \times 1 = 2$$

$$2 \times 4 = 8$$

$$5 \times 4 = 20$$

4. Next to the barn is a pen with 2 sheep and 3 ducks. How many legs altogether? 14 ✓

Show how you know your answer is correct.

$$2 + 4 + 3 + 3 = 14$$

5. One of the farmer's pens has a high fence around it. He can see 32 legs under the fence. How many sheep and ducks are in this pen?

9 ducks 6 sheep

Show one way to have sheep and ducks with 32 legs in all.

$$5 \overline{) 30} \quad 9 \overline{) 36}$$


Show a different number of sheep and ducks with 32 legs in all.

10 ducks 5 sheep


The work of Students P and Q are included as examples of incomplete or incorrect work toward proof. Both of these papers, along with the work of others, shows work toward a total of 32 but not necessarily 32 legs belonging to sheep and ducks.

Look at the work of Student P. While it is true that 3 tens will make 30, it is unrealistic in terms of sheep legs. *Did the student miswrite 24 in the 2<sup>nd</sup> part of 5? Did they mean that 14 ducks gives 28 legs? Why might we think this was an error in writing rather than one of addition or multiplication?*

**Student P**



### Sheep and Ducks



The farmer raises sheep and ducks.

1. How many legs on one duck? 2 ✓
2. How many legs on 4 ducks? 8 ✓
3. How many legs on 5 sheep? 20 ✓

4. Next to the barn is a pen with 2 sheep and 3 ducks. How many legs altogether? 14 ✓

Show how you know your answer is correct.

I added 4 sheep's legs it = 8  
and then I added the ducks  
legs together and it = 6 and I added /  
8 + 6 = 14 ✓

5. One of the farmer's pens has a high fence around it. He can see 32 legs under the fence. How many sheep and ducks are in this pen?

Show one way to have sheep and ducks with 32 legs in all.

I added ten sheep legs  
together to = 30 and added  
one duck to = 2 30 + 2 = 32

Show a different number of sheep and ducks with 32 legs in all.

I added 24 ducks and it = 28  
I added 1 sheep to = 4  
28 + 4 = 32

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Student Q was able to meet the demands of the first 4 parts of this task. In part 5 of the task, however, we see challenges. We are provided with evidence of several ways to get to 32 – including counting on from 15 but little acknowledgement of the reality of this part of the task. *Was this a function of having to work backwards from 32? Was this a matter of too many questions in this task for this student? Was it a matter of not having had on-going problem solving as part of the mathematics curriculum?*

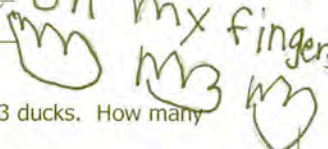
**Student Q**




**Sheep and Ducks**



The farmer raises sheep and ducks.

1. How many legs on one duck? 2 ✓ I counted!
2. How many legs on 4 ducks? 8 ✓ on my fingers
3. How many legs on 5 sheep? 20 ✓ 
4. Next to the barn is a pen with 2 sheep and 3 ducks. How many legs altogether? 14 ✓

Show how you know your answer is correct.

- 
5. One of the farmer's pens has a high fence around it. He can see 32 legs under the fence. How many sheep and ducks are in this pen? 22 10 32 I did an equation.   
 *it to how many I needed*   
 *I drew a picture of the legs doubled*

Show one way to have sheep and ducks with 32 legs in all.

$$10 + 10 + 5 + 6 + 1 = 32$$

I did an equation

Show a different number of sheep and ducks with 32 legs in all.

$$\begin{array}{r} 15 \\ + 17 \\ \hline 32 \end{array}$$

I counted on.



<b>Student Task</b>	Find the number of legs for a given number of sheep, of ducks and of sheep and ducks together. Use mathematical models and/or symbols to represent these situations. Find two combinations of sheep and ducks legs with 32 in all.
<b>Core Idea 3: Patterns, Functions, and Algebra</b>	<b>Understand patterns and use mathematical models to represent and to understand quantitative relationships.</b> <ul style="list-style-type: none"> <li>Describe, extend, and create patterns of number</li> </ul>
<b>Core Idea 2: Number Operations</b>	<b>Understand the meanings of operations and how they relate to each other, make reasonable estimates, and compute fluently.</b> <ul style="list-style-type: none"> <li>Understand situations that entail multiplication and division such as equal groupings of objects and equal sharing.</li> </ul>

*Mathematics of the task:*

- Ability to add and/or count with fluency and accuracy
- Ability to identify situations that entail multiplication such as equal groupings
- Ability to describe and extend patterns of number

*Based on teacher observation, this is what second graders knew and were able to do:*

- Name the number of legs on one duck and the number of legs on 4 ducks
- Draw to show the correct number of legs on 3 ducks and 2 sheep
- Draw or write a sequence of numbers to show one way to have 32 legs between some number of ducks and sheep

*Areas of difficulty for second graders:*

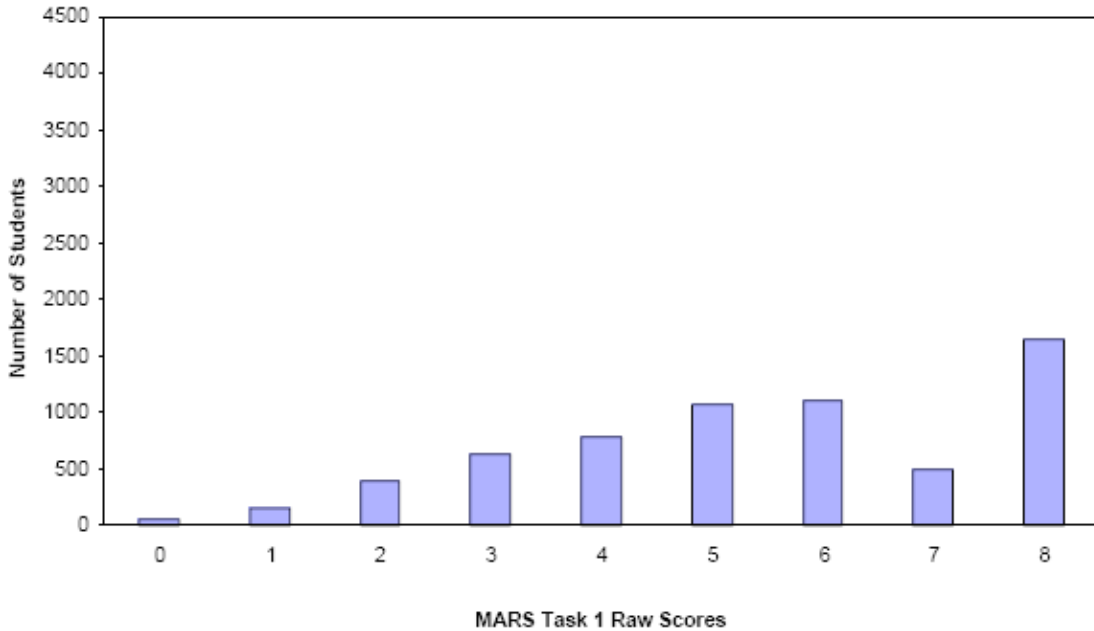
- Organizing their work
- Showing their work
- Working backwards from a given number of legs to create correct combinations of sheep and duck legs together

*Strategies used by successful students:*

- Using pictures or models as a problem solving strategy
- Using pictures with number facts
- Skip counting by 2s and 4s and then adding on with 2s and 4s
- Drawing 32 lines and circled groups of 2s and 4s

## Frequency Distribution for Task 1 – Grade 2 – Sheep and Ducks

### Sheep and Ducks Mean: 5.48      StdDev: 2.06



Score:	0	1	2	3	4	5	6	7	8
Student Count	46	146	396	625	774	1074	1102	494	1648
% ≤	0.7%	3.0%	9.3%	19.2%	31.5%	48.5%	66.0%	73.9%	100%
% ≥	100%	99.3%	97.0%	90.7%	80.8%	68.5%	51.5%	34.0%	26.1%

There is a maximum of 8 points for this task  
The cut score for a level 3 response, meeting standards, is 5 points.

Most students, 66%, were able to meet the essential demands of this task. A little more than one-quarter of the students were able to meet all the demands of the task including finding two different combinations of sheep and ducks with 32 legs in all. Less than 1% scored a zero on this task. All students attempted work on this task.

## Sheep and Ducks

Points	Understandings	Misunderstandings
<b>0 - 2</b>	The sample papers did not include any with scores of “0” for this task. Students could find the number of legs on one duck and many were able to find for four ducks.	Students scoring 2 and below had little access to problem solving. These papers were characterized by incorrect symbolic notations, incomplete drawings or in correct symbolic labeling of totals. In answering question 4, one-third of the students scoring a “2” gave “5” as an answer – counting the number of animals rather than the number of legs on the 5 animals.
<b>3 - 4</b>	Students could find the number of legs for one duck and four ducks and some were able to find the number of legs for five sheep. 67% of these students were successful finding both the answer and the solution path for question 4 – the number of legs on 2 sheep and 3 ducks. 33% of these students were able to find one way to have 32 sheep and duck legs using direct modeling and/or counting strategies.	15% of these papers continued to count the number of animals rather than the number of legs on 2 sheep and 3 ducks. Most unsuccessful attempts to find 32 legs in all attempted to manipulate symbols that were not based in multiples of 4s and 2s.
<b>5</b>	Four-fifths of these students could find the answers for the number of legs for one duck, 4 ducks and 5 sheep as well as for 2 sheep and 3 ducks. 50% of the students could find at least one combination of sheep and duck legs equal to 32.	Of the unsuccessful attempts to find a combination of sheep and duck legs equal to 32, errors consisted of miscounting, incomplete drawings, inaccurate calculations and unrealistic totals of legs (i.e. 17 sheep legs and 17 duck legs).
<b>6 - 7</b>	42% of all students scoring a 6 or a 7 were able to find 2 unique ways to have 32 sheep and duck legs. 82% were able to find at least one way to a combination of 32 legs.	Most errors in this scoring range were from miscalculating the number of legs for 4 ducks or for 5 sheep.
<b>8</b>	Students who met all the demands of this task did so using direct modeling, counting and addition strategies or recalling known facts.	



## Implications for Instruction:

It is essential that we give students opportunities to apply the mathematical concepts and the skills they are learning in problem solving situations. Young students can engage in substantive problem solving and in doing so actually develop basic skills, higher-order thinking skills and problem solving strategies. (Cobb et al 1991, Trafton & Hartman 1997) What better way for students to gain a greater appreciation for the power and beauty of mathematics!

### We can positively impact problem solving experiences if we keep these things in mind:

- **Use interesting and challenging problems**

A good problem to be solved should be interesting and motivating. The mathematics should be at the center of the problem and it should give them a chance to demonstrate what they know as it encourages them to use the mathematics they are currently learning. The best problems are not only interesting but they may also involve more than one mathematical concept and sometimes be a bit of ambiguity – in order to really challenge and encourage discussions.

- **Help students to clarify the problem to be solved**

Read the problem aloud together. This will remove any possible impediments due to reading skills. Ask students to restate the problem in their own words. Then have students ask any clarifying questions they might have. For the younger students, it will be necessary to model and practice asking good clarifying questions. *Are we trying to find out how many legs on 5 sheep or 5 ducks? Am I to find two different combinations of 32 legs or two ways to write a combination of 32 legs?*

- **Suggest questions students can ask when they get stuck on a problem**

Help students to begin to ask questions of themselves before, during and after solving a problem: *What am I trying to find out? What information do I already have? Am I missing any information? What strategies might I use? How can I represent my solution so that it shows I am correct?*

- **Create a classroom that allows for mistakes as well as successes**

On the way to being right – it is crucial that it be okay to be wrong. Problem solving is hard work and often frustrating. In problem solving situations people make mistakes but what is important is to keep on trying. A mistake can tell you information that might lead you to a correct second solution path. A few mistakes along the way might even help!

- **Encourage students to find and discuss strategies and solutions**

Two things will definitely happen any time you problem solve with your students – they will think of strategies and solution paths that did not occur to you and they will get answers wrong. In taking the time to discuss students' solutions after the problem solving event is beneficial to all students. Students who did the problem incorrectly are able to not only hear the right answer but also see how it was solved. Through the sharing of strategies we can assist students to hear and maybe try on new strategies and new techniques for solving future problems.

- **Give students many opportunities to solve interesting problems**

The way to improve problem solving skills is to solve lots of problems. Students will learn new problem-solving strategies, how to ask for clarification, ways to learn from mistakes, and the importance of perseverance, tenacity, and flexibility. Problem solving provides opportunities to apply the math they are learning in new and challenging context. It helps us to define what it really means “to do” mathematics.

### **Ideas for Action Research:**

Review you curriculum materials and other problem-solving resources with the following questions in mind:

- Is the task expandable? Does it allow entry for all? Is it challenging for all?
- Is the problematic or engaging aspect of this task the mathematics?
- Does the task require explanation and justification?
- Will the student learn from the task and will I learn more about each student from the attempt?

After completing the problem with your students, reflect on the focus of the discussion and their written communication. Carefully consider the evidence you have gathered during the problem solving process, the class discussion and from the written work you gather.

- In what ways were students able to engage in the task? What adjustments needed to be made along the way?
- Were students’ thinking and reasoning evident in their work? Why or why not?
- Were all students actively engaged in discussions – during the solving process, during the sharing and comparing strategies time?
- Were students sharing both how and why their methods work? Were students able to convince others of the correctness of their solutions?
- What happened when a mistake was shared? What questions did students ask? What questions might have clarified discrepancies?

Reflect on these questions periodically to deepen your students’ problem solving experiences and to strengthen your problem solving curriculum.

Performance Assessment Task
Sheep and Ducks Grade 2
This task challenges a student to use addition and multiplication to think about and calculate equal-sized groups. Students need to be able to problem-solve about strategies for sharing legs between sheep (groups of 4) and ducks (groups of 2). Students need to be able to use words, pictures, and numbers to explain how to parcel out the legs between the 2 groups.
Common Core State Standards Math - Content Standards
<p><b><u>Operations and Algebraic Thinking</u></b>  <b>Represent and solve problems involving addition and subtraction.</b>  2.OA.1 Use addition and subtraction with 100 to solve one- and two-step word problems involving situations of adding to, taking from, putting together, taking apart, and comparing with unknowns in all positions, e.g. by using drawings and equations with a symbol for the unknown number to represent the problem.</p> <p><b>Work with equal groups of objects to gain foundations for multiplication.</b></p> <p><b><u>Number and Operations in Base Ten</u></b>  <b>Use place value understanding and properties of operations to add and subtract.</b>  2.NBT.5 Fluently add and subtract within 100 using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction.</p>
Common Core State Standards Math – Standards of Mathematical Practice
<p><b>MP.3 Construct viable arguments and critique the reasoning of others.</b>  Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and – if there is a flaw in an argument – explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.</p> <p><b>MP.4 Model with mathematics.</b>  Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts, and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.</p>

Assessment Results				
This task was developed by the Mathematics Assessment Resource Service and administered as part of a national, normed math assessment. For comparison purposes, teachers may be interested in the results of the national assessment, including the total points possible for the task, the number of core points, and the percent of students that scored at standard on the task. Related materials, including the scoring rubric, student work, and discussions of student understandings and misconceptions on the task, are included in the task packet.				
Grade Level	Year	Total Points	Core Points	% At Standard
2	2007	8	5	68.5 %